# A2 Maths Summer Task 2018

The following questions need to be answered on A4 lined/square paper showing working out where necessary and submitted in a plastic wallet to your teacher during your <u>first lesson in September.</u>

You must ensure you do not leave questions unanswered. If you do not know how to complete a question then use the online textbook, mathswebsite.com or other online sources to help you.

It is extremely important that you are confident with the topics in this pack as the first half term of A2 Maths is extremely fast paced. The topics in the pack form a large part of the A2 course so you will be at a **big** disadvantage if you are not prepared for them.


Mrs A. Collier

2i/c Mathematics with Responsibility for KS5

# Proof by contradiction

(i)     Disprove the following statement

$$\text{If } p > q, \text{ then } \frac{1}{p} < \frac{1}{q}$$

[2]

(ii)    State a condition on p and q so that the statement is true.

[1]

**Question 2**

Positive integers a, b and c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$.

(i)     Given that t is an integer greater than 1, show that 2t, $t^2 - 1$ and $t^2 + 1$ form a Pythagorean triple.

[3]

(ii)    The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.
Use this triple to show that not all Pythagorean triples can be expressed in the form 2t, $t^2 - 1$ and $t^2 + 1$.

[4]

**Question 3**

(i)     Disprove the following statement

$$3^n + 2 \text{ is prime for all integers } n \geq 0$$

[4]

(ii)    Does any number of the form $3^n$ , where n is a positive integer, have 5 as its final digit?  Justify your answer.

[6]

# Simplifying rational expressions
**Question 1**

Simplify      $\frac{1}{2+\sqrt{x}}$

[3]

**Question 2**

Simplify      $\frac{3x-2}{9x^2-4}$

[3]

**Question 3**

Simplify      $\frac{3x^2+7x+4}{6x^2-x-12}$

[4]

# Partial fractions

### Question 1

Express $\frac{1}{(x+5)(x-4)}$ in partial fractions

[4]

### Question 2

Express $\frac{4}{x(x^2+4)}$ in partial fractions

[6]

### Question 3

Given that

$$\frac{3+2x^2}{(1+x)^2(1-4x)} = \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{1-4x}$$

Where A, B and C are constants, find B and C and show that A = 0

[4]

### Question 4

Express $\frac{3x+2}{x(x^2+1)}$ in partial fractions

[6]

# Binomial expansion for rational n.

### Question 1
Find the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{1+2x^3}}$ , and state the set of values of x for which the expansion is valid.

[5]

### Question 2
Find the first three terms of the binomial series expansion of $\sqrt[3]{1+2x}$ , and state the set of values of x for which the expansion is valid.

[5]

### Question 3
Find the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{1+4x^2}}$ , and state the set of values of x for which the expansion is valid.

[5]

### Question 4
Find the first four terms of the binomial series expansion of $\sqrt{4+2x}$ , and state the set of values of x for which the expansion is valid.

[5]

# Moments of a force

## Question 1

A plank AB has mass 12 kg and length 2.4 m.  A load of mass 8 kg. is attached to the plank at the point C, where AC = 0.8 m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B as in the diagram below.

The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.
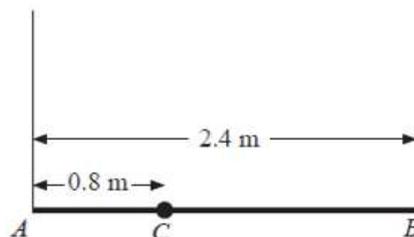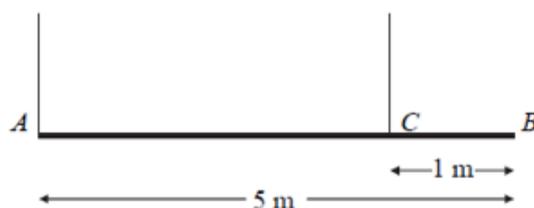


(i)     Find the tension in the rope attached to B.

[4]

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 10 N greater than the tension in the rope attached at B.

(ii)    Find the distance of the centre of mass of the plank from A.

[6]

## Question 3

A beam AB has mass 12 kg. and length 5 m.  It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam.  One rope is attached to A, the other to the point C on the beam, where BC = 1 m. as shown in the diagram.  The beam is modelled as a uniform rod, and the ropes as light strings.



(i)     Find the tension in the rope at C

[3]

(ii)    Find the tension in the rope at A

[2]

## Question 4

A uniform ladder of length 8 m and weight 180 N rests against a smooth, vertical wall and stands on a rough horizontal surface.  A woman of weight 720 N stands on the ladder so that her weight acts at a distance $x$ m from its lower end, as shown in the diagram.
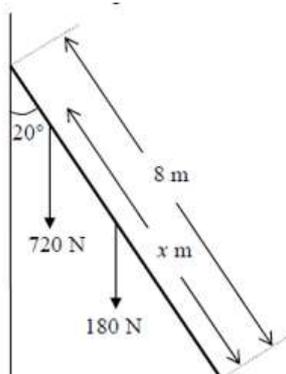


The system is in equilibrium at 20° to the vertical.  Show that the frictional force between the ladder and the horizontal surface is F N, where F = 90 (1 + $x$) tan 20

[5]